

Comment on "The Tropospheric Land-Sea Warming Contrast as the Driver of Tropical Sea Level Pressure Changes" by Bayr and Dommenges

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Abstract

Bayr and Dommenges [2013] proposed a model of temperature-driven air redistribution to quantify the ratio between changes of sea level pressure p_s and mean tropospheric temperature T_a in the tropics. This model assumes that the height of the tropical troposphere is isobaric. Here problems with this model are identified. A revised relationship between p_s and T_a is derived governed by two parameters – the isobaric and isothermal heights – rather than just one. Further insight is provided by the model of Lindzen and Nigam [1987] which was the first to use the concept of isobaric height to relate tropical p_s to air temperature, and did this by assuming that isobaric height is always around 3 km and isothermal height is likewise near constant. Observational data, presented here, show that neither of these heights is spatially universal nor do their mean values match previous assumptions. Analyses show that the ratio of the long-term changes in p_s and T_a associated with land-sea temperature contrasts in a warming climate – the focus of Bayr and Dommenges [2013] – is in fact determined by the corresponding ratio of spatial differences in the annual mean p_s and T_a . The latter ratio, reflecting lower pressure at higher temperature in the tropics, is dominated by meridional pressure and temperature differences rather than by land-sea contrasts. Considerations of isobaric heights are shown to be unable to predict either spatial or temporal variation in p_s . As noted by Bayr and Dommenges [2013], the role of moisture dynamics in generating sea level pressure variation remains in need of further theoretical investigations.

1 Introduction

Low-level tropical winds are generally linked to convection, but the physical processes and relationships remain a matter of interest and discussion. Indeed, our incomplete understanding of the physical principles governing low-level circulation is manifested by the inability of atmospheric models to replicate the terrestrial water cycle [Marengo, 2006, Hagemann et al., 2011] as well as by the challenge of confidently predicting precipitation and air circulation [e.g., An, 2011, Acharya et al., 2011, Huang et al., 2013]. One recurring question is whether the release of latent heat in the upper atmosphere generates sufficient moisture convergence in the lower atmosphere to feed convection. The observed relationship between sea level pressure and surface temperature (with warm areas having low pressure) is regarded as evidence

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that low-level convergence is, rather, driven by the temperature gradients [see discussions by Lindzen and Nigam, 1987, Neelin, 1989, Sobel and Neelin, 2006, Back and Bretherton, 2009, An, 2011].

The physical rationale behind surface pressure gradients driven by surface temperature gradients is that a gaseous atmosphere held by a gravitational field cannot remain static in the presence of a horizontal temperature gradient [Landau and Lifshitz, 1987]. Any differential heating causes pressure differences in the upper atmosphere to arise due to the larger exponential scale height (pressure gradient by altitude) of a warmer versus a colder atmospheric column. As illustrated by the Bjerknes circulation theorem [Thorpe et al., 2003], this implies circulation: upper-level air divergence from the warmer air column and low-level convergence towards it. However, to estimate the strength of this circulation requires a shift from qualitative to quantitative considerations.

The magnitude of the surface pressure gradient can be found if one knows the isobaric height – the altitude where pressure does not vary over space. Where temperature is high (and air density is low) there is less air below the isobaric height than where temperature is low (and air density is high) – this follows from the hypsometric equation, which captures the hydrostatic equation and the ideal gas law. Accordingly, the weight of the air column is lower in warmer than in colder areas. The resulting surface pressure and temperature gradients can be shown to be proportional to each other, with the proportionality coefficient depending on the isobaric height.

It appears that if we could determine isobaric height from some independent considerations we could use air temperature to predict surface pressures. To address this challenge Bayr and Dommenget [2013] proposed a simple physical model of temperature-driven air redistribution which they claimed to satisfactorily quantify the relationship between tropical sea level pressure p_s and the mean tropospheric temperature T_a under the assumption that the isobaric height z_e is the height of the troposphere, $z_e = 16.5$ km. However, while not cited by Bayr and Dommenget [2013], in an earlier influential study Lindzen and Nigam [1987] suggested that a similar relationship between p_s and surface temperature T_s can be quantitatively explained assuming that the tropical isobaric height is around 3 km. This discrepancy requires a discussion, because with $z_e = 3$ km the model of Bayr and Dommenget [2013] no longer agrees with observations.

Another point is that to obtain a satisfactory agreement with the data in their model Lindzen and Nigam [1987] had to use an additional parameter besides the isobaric height. This additional parameter describes how fast surface temperature differences diminish with altitude thereby defining a certain approximately isothermal height where no information about the surface temperature contrasts is preserved. This isothermal height describes horizontal variation in the vertical lapse rate of air temperature. Mean tropospheric temperature investigated by Bayr and Dommenget [2013] should be clearly affected by such variation. Bayr and Dommenget [2013] neglected any spatial variability in lapse rate in their model.

Here we re-examine the derivation of Bayr and Dommenget [2013] to identify and resolve several inconsistencies in their model (Section 2). We derive a general relationship linking the ratios of horizontal differences of surface pressure and air temperature (surface and mean tropospheric) to an isobaric height. We show that, in agreement with the model of Lindzen and Nigam [1987] and in contrast with the model of Bayr and Dommenget [2013] who consider only isobaric height, these ratios are a function of two heights, isobaric and isothermal (Section 3).

Using data provided by the National Centers for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) Reanalysis [Kalnay et al., 1996] and the Remote Sensing Systems [Mears and Wentz, 2009] we then assess the isobaric and isothermal heights in the tropics (Section 4). We demonstrate in theory that the relationship between sea level pressure and T_a is significantly more sensitive to any changes in isothermal height than is the relationship between p_s and T_s . Accordingly, the ratio $\Delta p_s / \Delta T_a$ is not constant in the tropics and increases by about a factor of three from the higher latitudes towards

the equator. Meanwhile the $\Delta p_s/\Delta T_s$ is more spatially stable (Section 5). Our analysis of the data further reveals that neither of the two assumptions made by Lindzen and Nigam [1987] concerning the isobaric and isothermal heights appears plausible. The isobaric height is highly variable with a different distribution for land and ocean. The isothermal height is also spatially variable. We show that this variability does not allow one to estimate the relationship between sea level pressure and temperature from the values of isobaric and isothermal heights with any certainty (Section 6).

We then discuss temporal versus spatial variability of sea level pressure and air temperature (Section 7). While this distinction was not clearly drawn by Bayr and Dommenget [2013], we show that their data reveal an interesting pattern. The observed long-term temporal changes of sea level pressure and mean tropospheric temperature are characterized by the same ratio as their respective spatial changes. In both cases land displays a smaller by absolute magnitude ratio than the ocean. This pattern matches the observations but is not reproduced in the multimodel ensemble of the Intergovernmental Panel on Climate Change (IPCC).

We conclude with a discussion of possible directions for future research as to how consideration of the relationships between sea level pressure and air temperature could inform our understanding of the principles governing low-level atmospheric circulation and moisture convergence (Section 8).

2 The model of Bayr and Dommenget [2013]

Bayr and Dommenget [2013] begin their derivation with an equation they refer to as "the hydrostatic equation"

$$dp = -\rho g d\eta \quad (1)$$

with pressure p , density ρ , gravity constant g , and η described as "air column height"¹. According to Bayr and Dommenget [2013], for an "isobaric thermal expansion of the air column" it follows from the ideal gas law that

$$d\eta = \frac{\eta}{T} dT, \quad (2)$$

where T is temperature. They conclude that using Eqs. (1) and (2) one obtains how sea level pressure depends on temperature

$$\frac{dp}{dT} = \frac{1}{2} \rho g \frac{\eta}{T}. \quad (3)$$

We first note that both the $1/2$ multiplier and the lack of the minus sign in (3) are not consistent with (1) and (2). Bayr and Dommenget [2013] explain the appearance of $1/2$ using a graphical scheme that we have re-drawn in Fig. 1. They explain that "to balance the heights of the two columns at the end, half of the height difference is moved from the warmer to the colder air volume". As we can see from Fig. 1, this statement refers to the difference in heights η between two local columns. However, to test their model against the data, Bayr and Dommenget [2013] define dp and dT in (3) to represent the relative changes $d(p_s - \overline{p_s})$ and $d(T_a - \overline{T_a})$, where p_s and T_a are the local values of sea level pressure and mean tropospheric temperature, respectively, and the overbar their mean values in the tropics. If the mean values $\overline{\eta}$, $\overline{p_s}$ and $\overline{T_a}$ change negligibly in time compared to their local values (i.e. $d\overline{X} \approx 0$ with $X = p_s, T_a, \eta$), replacement of variables $X \rightarrow X - \overline{X}$ does not affect Eqs. (1) and (2). But it does impact the balancing procedure in Fig. 1. Indeed, to balance height η between the two columns one has to move not one half but the *entire* difference $d(\eta - \overline{\eta}) = (1/2)(\eta_1 - \eta_2)$ from the warmer to the colder column. Therefore, if by dp/dT in (3) one understands, as do Bayr and Dommenget [2013], relative changes of the respective

¹In the derivation of Bayr and Dommenget [2013] η in (1) is denoted as h .

variables, one has no grounds to introduce the 1/2 multiplier into Eq. (3) (see Eq. (20) in the next section).

The sign discrepancy between (1) and (3) appears as a simple error, but in fact it manifests the misapplication of Eq. (1). This equation is not a hydrostatic equation and contradicts the latter. Let us illustrate this point. For atmospheric air conforming to the ideal gas law

$$p = NRT, \quad R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}, \quad (4)$$

where N is molar density, the hydrostatic equilibrium equation is

$$\frac{\partial p(z)}{\partial z} = -\rho(z)g = -\frac{p}{h}, \quad h \equiv \frac{RT(z)}{Mg}, \quad (5)$$

where M is molar mass and z is height above the sea level, p , ρ and T are local values of pressure, density and temperature at height z . The hydrostatic equilibrium equation (5) says nothing about temporal changes of either pressure or temperature. It only describes the distribution of air pressure with height.

In Eqs. (1)-(3) Bayr and Dommenges [2013] interpreted pressure p as sea level pressure $p = p_s$, density ρ as the mean air density in the troposphere $\rho = \rho_a = 0.562 \text{ kg m}^{-3}$ and air column height η as the height $H = 16.5 \text{ km}$ of the tropical troposphere corresponding to height η_{100} of pressure level $p_{100} = 100 \text{ hPa}$. They also interpreted differentials in (1)-(3) as describing temporal changes of the corresponding variables. From (5) we find that sea level pressure p_s is related to ρ_a and η_{100} as follows:

$$p_s \equiv \int_0^\infty \rho g dz = \rho_a g \eta_{100} + p_{100}, \quad \rho_a \equiv \frac{1}{\eta_{100}} \int_0^{\eta_{100}} \rho(z) dz. \quad (6)$$

Taking differential of (6) we obtain

$$dp_s = \rho_a g d\eta_{100} + g \eta_{100} d\rho_a. \quad (7)$$

When we compare (7) and (1) with $p = p_s$, $\rho = \rho_a$ and $\eta = \eta_{100}$ it is apparent that in (1) the minus sign was incorrectly added to the first term in (7). The second term in (7), compressibility of the atmospheric air $d\rho_a \neq 0$, was dropped altogether.

For an incompressible fluid with $\rho = \rho_a = \text{constant}$ Eq. (7) has the familiar meaning relating column height to surface pressure: the larger the height of the fluid column, the higher the surface pressure. Meanwhile the minus sign in Eq. (1) presumes exactly the opposite: the larger the column height η , the smaller the surface pressure. On the other hand, if Bayr and Dommenges [2013], having ignored air compressibility for an unknown reason, used Eq. (1) with the plus sign, the sign discrepancy between (3) and (1) would have disappeared. However, in this case Eq. (3) would have yielded a positive relationship of surface pressure on temperature (higher pressure at larger temperature) which contradicts the observations: in the real atmosphere lower sea level pressure is associated with higher temperature.

We conclude that as it is based on an incorrect equation (1) the model of Bayr and Dommenges [2013] lacks any credible quantitative explanatory power and yields values similar to observations only by chance.

3 Dependence of sea level pressure on temperature

We will here derive a general relationship linking surface pressure and temperature to the vertical structure of the atmosphere. The model of Bayr and Dommenges [2013] did not consider how temperature might vary with height. We will allow air temperature to vary with height with a lapse rate $\Gamma \equiv -\partial T/\partial z$, which is independent of height but can vary in the horizontal direction.

We introduce the following dimensionless variables to replace height z and lapse rate Γ :

$$Z \equiv \frac{z}{h_s}, \quad c \equiv \frac{\Gamma}{\Gamma_g}, \quad h_s \equiv \frac{RT_s}{Mg} \equiv \frac{T_s}{\Gamma_g}, \quad \Gamma_g \equiv \frac{Mg}{R} = 34 \text{ K km}^{-1}, \quad M = 29 \text{ g mol}^{-1}. \quad (8)$$

Here Γ_g is the so-called autoconvective lapse rate. For air temperature we have

$$T(Z) = T_s(1 - cZ), \quad Z < c^{-1}, \quad T_s \equiv T(0). \quad (9)$$

In these variables the hydrostatic equilibrium equation (5) assumes the form

$$-\frac{\partial p}{\partial Z} = \rho g h_s = \frac{p}{1 - cZ}. \quad (10)$$

Solving (10) for $p \geq 0$ we have

$$\ln \frac{p}{p_s} = - \int_0^Z \frac{dZ'}{1 - cZ'} = \frac{1}{c} \ln(1 - cZ) \approx -Z - \frac{1}{2}cZ^2. \quad (11)$$

The approximate equality in (11) holds for $cZ \ll 1$, which for $\Gamma \approx 6 \text{ K km}^{-1}$ corresponds to $z \ll h_s(\Gamma_g/\Gamma) \approx 50 \text{ km}$. This is always the case in the troposphere.

Pressure $p(z)$ and temperature $T(z)$ at a given height z are functions of p_s , T_s and Γ . Considering linear deviations from the mean values of $\overline{p_s}$, $\overline{T_s}$, and $\overline{\Gamma}$ and taking the total differential of the approximate relationship for p (11) over these three variables we obtain:

$$dp = p_s(da + Zdb - \frac{1}{2}Z^2dc)e^{-Z}, \quad (12)$$

where da , db and dc stand for the dimensionless differentials of p_s , T_s and Γ :

$$da \equiv \frac{dp_s}{p_s} \approx \frac{dp_s}{\overline{p_s}}, \quad db \equiv \frac{dT_s}{T_s} \approx \frac{dT_s}{\overline{T_s}}, \quad dc \equiv \frac{d\Gamma}{\Gamma_g}, \quad (13)$$

where $\overline{p_s} = 1013 \text{ hPa}$ and $\overline{T_s} = 298 \text{ K}$ are the annual mean sea level pressure and surface air temperature in the tropics. The inaccuracy of the approximate relationships in (13) is determined by the relative changes of sea level pressure and surface temperature across the tropics. For the zonally averaged p_s and T_s this inaccuracy does not exceed 4%.

Isothermal height $z_i \equiv Z_i h_s$ is found by taking total differential of T (9) over T_s and Γ and putting $dT = dT_s - T_s Z_i dc = 0$. This gives

$$Z_i = \frac{db}{dc} = \frac{1}{h_s} \frac{dT_s}{d\Gamma}, \quad z_i \equiv Z_i h_s = \frac{dT_s}{d\Gamma}. \quad (14)$$

We can see from (14) that an isothermal height exists if only $db/dc > 0$, i.e. if areas with a warmer surface have a higher lapse rate. Bayr and Dommenges [2013] note that this pattern should be related to moisture availability. They note that, below an isothermal surface, drier areas should have a steeper lapse rate close to dry adiabat and thus get warmer than moist areas where the lapse rate is lower because of latent heat release. In the tropical atmosphere, moist areas, most notably the equatorial regions, have on average a steeper mean tropospheric lapse rate than do the drier regions at higher tropical latitudes (Fig. 2). In the lower atmosphere this has to do with the trade wind inversion, also mentioned by Bayr and Dommenges [2013], which is mostly pronounced in the drier (colder) regions where the lapse rate in the low atmosphere is very small. In the upper troposphere (around the isothermal height) a higher lapse rate in the moister regions is due to the fact that in such regions the air ascends rapidly and thus has a lapse rate more close to adiabatic than in the slowly descending air, where a more significant part of the thermal energy can be radiated to space. It is only in the middle atmosphere that, because of latent heat release, the lapse rate

over the moist equatorial areas is smaller than it is at higher tropical latitudes. Generally, both in the lower atmosphere and on average in the troposphere $db/dc > 0$ is fulfilled.

Isobaric height $z_e \equiv Z_e h_s$ is defined from (12) as the height where $dp = 0$. It is determined from the following quadratic equation:

$$da + Z_e db - \frac{1}{2} Z_e^2 dc = 0, \quad Z_e = Z_i \left(1 \pm \sqrt{1 + \frac{2}{Z_i} \frac{da}{db}} \right). \quad (15)$$

There can be two isobaric heights (Fig. 3). Note that the isobaric height Z_e (15) does not depend on lapse rate c but only on its differential dc via Z_i . This is a consequence of the smallness of $cZ \ll 1$ in the troposphere.

From (15) and (14) we obtain the following relationship for the ratio of the differentials of surface pressure and temperature (13):

$$\frac{da}{db} = -Z_e \left(1 - \frac{1}{2} \frac{Z_e}{Z_i} \right). \quad (16)$$

When $db = 0$, i.e., when the surface temperature does not vary, but only lapse rate does, we have from (15)

$$\frac{da}{dc} = \frac{Z_e^2}{2}. \quad (17)$$

The surface pressure change is proportional to the change in lapse rate, i.e. the pressure is lower where the lapse rate is smaller, with the proportionality coefficient equal to half the squared isobaric height.

To find the relationship between the isobaric height and mean tropospheric temperature T_a below the 100 hPa level we need to find the relationship between T_a and T_s . This relationship takes the form (see Appendix):

$$\frac{db}{dn} = \frac{1}{1 - 0.66/Z_i}, \quad dn \equiv \frac{dT_a}{T_a}. \quad (18)$$

Finally from (16) and (18) we obtain:

$$\frac{da}{dn} = -Z_e \left(1 - \frac{1}{2} \frac{Z_e}{Z_i} \right) \frac{1}{1 - 0.66/Z_i}. \quad (19)$$

Relationships (16) and (19) are shown in Fig. 4.

When, as in the model of Bayr and Dommenguet [2013], lapse rate is assumed to be constant with $dc = 0$, we have $Z_i = \infty$ and (19), using notations (13) and $dn \equiv dT_a/T_a$, becomes

$$\frac{da}{db} = \frac{da}{dn} = -Z_e, \quad \frac{dp_s}{dT_a} = -\frac{z_e}{h_s} \frac{p_s}{T_a} = -\rho_s g \frac{z_e}{T_a}. \quad (20)$$

Comparing (20) to (3) of Bayr and Dommenguet [2013] we notice the absence of coefficient $1/2$ in (20) and the presence of surface air density ρ_s in (20) instead of mean tropospheric air density $\rho = \rho_a$ in (3). If the lapse rate did not vary in the horizontal plane, Eq. (20) would be the correct equation relating ratio of pressure and temperature differences to an isobaric height. But as we will show below in the real atmosphere the lapse rate variation cannot be neglected.

Considering $dp = \Delta p(z)$ in (12) as a small pressure difference at a given height between two air columns, we note that this difference has a maximum above the isobaric height Z_e (15) at a certain height Z_0 . This height is determined by taking the derivative of (12) over Z and equating it to zero, see (12), (15) and (14):

$$\frac{\partial \Delta p}{\partial Z} = 0, \quad da + Z_0 db - \frac{1}{2} Z_0^2 dc - db + Z_0 dc = 0, \quad Z_0 = 1 + Z_i \pm \sqrt{(Z_e - Z_i)^2 + 1}. \quad (21)$$

At this height the pressure difference is equal to

$$\Delta p_0 \equiv \Delta p(Z_0) = p_s e^{-Z_0} \left(da + Z_0 db - \frac{1}{2} Z_0^2 dc \right) = p_s e^{-Z_0} (db - Z_0 dc). \quad (22)$$

Note that by definition when $\Delta p_0 = 0$ we have $Z_e = Z_0 = Z_i$. As is clear from Fig. 3, where the vertical profiles of $\Delta p(z)$ (12) are shown for different values of da , db and dc , Δp_0 is the maximum pressure difference between the air columns above the lower isobaric height.

As follows from Eqs. (21), (22) and (16), height Z_0 as well as the ratio between the pressure surplus aloft and the pressure shortage at the surface $\Delta p_0/\Delta p_s$ are functions of two parameters, the isobaric and isothermal heights Z_i and Z_e . When Z_i and Z_e are constant, the ratio between the pressure surplus aloft and the pressure shortage at the surface in the warmer column is constant as well: the larger the pressure surplus aloft, the larger the surface pressure shortage, with a direct proportionality between the two. This is consistent with the conventional thinking about differential heating, that the upper pressure surplus causes air to diverge from the warmer column, the total amount of gas will diminish and there appears a shortage of pressure at the surface $\Delta p_s < 0$ [e.g., Pielke, 1981, Fig. 2].

When the vertical lapse rate is constant, $dc = 0$, from (21) we have $Z_0 = 1 - da/db$. In this case, as is clear from (22), for small values of $da/db \ll 1$ the magnitude of Δp_0 does not depend on da , but is directly proportional to db , i.e. to ΔT_s (13) (Fig. 3a). This means that under these particular conditions a surface temperature gradient directly determines the pressure gradient *in the upper atmosphere*. In this sense there is no difference between surface temperature gradient and a gradient of lapse rate related to latent heat release – both can only determine a pressure surplus aloft, cf. Fig. 3a,b. We emphasize that while under certain assumptions the magnitude of the tropospheric pressure gradient can be approximately specified from considerations of the hydrostatic balance and air temperature gradients alone, the magnitude of the surface pressure gradient cannot.

Generally, ratios da/db and db/dc in (15) and (14) can be understood as the ratios of the gradients of the corresponding variables, e.g. $da/db = (\partial p_s/\partial y)/(\partial T_s/\partial y)(T_s/p_s)$, where $(\partial p_s/\partial y)/(\partial T_s/\partial y)$ is the ratio of sea level pressure and surface temperature gradients in a given y direction (e.g. along the meridian). In this case for any y the value of z_e (or z_i) has the meaning of a height where $\partial p/\partial y = 0$ (or $\partial T/\partial y = 0$), i.e. where pressure (or air temperature) does not vary over y . These ratios can also be understood as the ratios of small finite differences between pressure or temperature in a given grid point and a certain reference value of pressure or temperature, $dp_s/dT_s = \Delta p_s/\Delta T_s$. This approach was taken by Lindzen and Nigam [1987] and Bayr and Dommenges [2013]. We can now estimate all parameters in (19) from empirical data to compare them with model assumptions.

4 Data

We used NCAR-NCEP reanalysis data on sea level pressure and surface air temperature, as well as on geopotential height and air temperature at 13 pressure levels provided by the NOAA/OAR/ESRL PSD, Boulder, Colorado, USA, from their Web site at <http://www.esrl.noaa.gov/psd/> [Kalnay et al., 1996]. As an estimate of the mean tropospheric temperature we took TTT (Temperature Total Troposphere) MSU/AMSU satellite data provided by the Remote Sensing Systems from their Web site at <http://www.remss.com/measurements/upper-air-temperature> [Mears and Wentz, 2009]. Monthly values of all variables were averaged over the time period from 1978 (the starting year for the TTT data) to 2013 to obtain 12 mean monthly values and one annual mean for each variable for each grid point on a regular $2.5^\circ \times 2.5^\circ$ global grid.² The following pressure levels covering the tropical troposphere were considered: 1000, 925, 850, 700, 600, 500, 400,

²TTT data array contains 144 (360/2.5) longitude and 72 (180/2.5) latitude values each pertaining to the center of the corresponding grid point. NCAR-NCEP data arrays contain 144 longitude and 73 latitude

300, 250, 200, 150, 100 and 70 hPa. Meridional gradients $\partial X/\partial y$ of variable X ($X = p_s, T_s$) at latitude y were determined as the difference in X values at two neighboring latitudes and dividing by 2.5° : $\partial X(y)/\partial y \equiv [X(y + 1.25^\circ) - X(y - 1.25^\circ)]/2.5^\circ$. Local pressure differences corresponding to pressure level p_j were calculated from the geopotential height differences $\Delta p_j = (z_j - \bar{z}_j)p_j/h_j$, where z_j is the local geopotential height of pressure level p_j , \bar{z}_j is its mean value in the considered spatial domain, $h_j = RT_j/(Mg)$ is the local exponential pressure scale height (5) and T_j is local air temperature at this level.

5 Spatial patterns

Our regression of the annual mean values of $\Delta p_s \equiv p_s - \bar{p}_s$ on $\Delta T_a \equiv T_a - \bar{T}_a$ (the overbars denote spatial averaging) for the tropical area between 22.5°S and 22.5°N produced a slope of $r = -2.3 \text{ hPa K}^{-1}$ with $R^2 = 0.75$ (Fig. 5a). This is practically identical to the results of Fig. 3 of Bayr and Dommenges [2013], where Δp_s and ΔT_a values for the four seasons are plotted together. The resulting regression slope $r = -2.4 \text{ hPa K}^{-1}$ with $R^2 = 0.76$ was interpreted by Bayr and Dommenges [2013] as describing seasonal changes of sea level pressure and tropospheric temperature. However, as our result shows, even if seasonal changes of $\Delta p_s/\Delta T_a$ were completely absent, the corresponding regression for the four seasons combined would nevertheless produce a non-zero slope reflecting the time-invariable spatial association between higher temperature and lower pressure in the tropics. The agreement between our relationship for the annually averaged Δp_s and ΔT_a ratio and the one shown in Fig. 3 of Bayr and Dommenges [2013] indicates that either the spatial variation dominates the seasonal changes or that the seasonal changes are, on average, characterized by a similar $\Delta p_s/\Delta T_a$ ratio as the spatial changes. Bayr and Dommenges [2013] did not discuss whether their Fig. 3 actually characterizes spatial or temporal variation but interpreted the results of their Fig. 3 as a test of validity of their model which they later used to assess long-term temporal changes in p_s and T_a .

Next we note that since a linear regression minimizes the departure of the empirical points from the theoretical line, the regression slope can be disproportionately influenced by the values that depart most from the mean. This depends on the shape of the frequency distribution of data points around the mean. For data points in Fig. 5a with their Δp_s departing from the zero mean by more than two thirds of standard deviation the regression slope is -2.5 hPa K^{-1} with $R^2 = 0.85$, which is very close to the overall regression (cf. Fig. 5a,b). Meanwhile the regression slope for the remaining grid points with smaller $|\Delta p_s|$ is -1.4 hPa K^{-1} with $R^2 = 0.30$ (Fig. 5c). This subset constitutes about half of all the points but harbors two thirds of all land values (Fig. 5d). This subset apparently makes a negligible contribution to the pantropical regression which, in consequence, appears uninformative with regard to a large portion of the data, including most land.

We further found that the regression slope r depends strongly on the averaging domain: it increases by absolute magnitude towards the equator (Fig. 6a). As r decreases with diminishing tropical area, so does the squared correlation coefficient (Fig. 6b), although for the oceanic grid points it remains relatively high even near the equator. E.g., for the area between 30°S and 30°N for the ocean we have $r = -1.5 \text{ hPa K}^{-1}$ with $R^2 = 0.75$ while for 5°S and 5°N we have $r = -4.2 \text{ hPa K}^{-1}$ with $R^2 = 0.64$. These patterns testify that the spatial relationship between Δp_s and ΔT_a is not universal in the tropics. In contrast, while similar regressions of Δp_s on surface temperature ΔT_s are characterized by lower R^2 ,

values each pertaining to the border of the corresponding grid point. E.g., the northernmost latitude in the NCAR-NCEP data is 90°N , while for the TTT data it is $90 - 2.5/2 = 88.75^\circ\text{N}$. This discrepancy was formally resolved by adding an empty line to the end of the TTT data such that the number of lines match and matching i, j grid points in the two arrays. In the result, every TTT value refers to a point in space that is 1.25 degree to the South and to the East from the coordinate of the corresponding NCAR-NCEP value. This relatively small discrepancy did not appear to have any impact on any of the resulting quantitative conclusions (i.e. if instead one moves TTT points to the North, the results are unchanged).

the regression slope, around -1 hPa K^{-1} , does not appear to depend significantly on the averaging domain (Fig. 6d,e).

To explore whether the relationship between Δp_s and ΔT_a is dominated by zonal or meridional differences, we performed a regression of zonally averaged Δp_s on zonally averaged ΔT_a for the area between 22.5°S and 22.5°N for different months (Fig. 6c). Zonally averaged values account for a major part of the dependence between Δp_s and ΔT_a : regression of annual mean zonally averaged values yields $r = -2.0 \text{ hPa K}^{-1}$. The same is true for the surface temperature (Fig. 6f). Since land/sea contrasts in the tropics are predominantly zonal, this means, again, that either the contribution of land/sea pressure/temperature contrasts to the pantropical regressions of Δp_s on ΔT_a and ΔT_s is relatively unimportant or that these contrasts are characterized by approximately the same ratio as the zonally averaged values.

6 Isobaric and isothermal heights

While the models of Lindzen and Nigam [1987] and Bayr and Dommenget [2013] build upon the notion of an isobaric height, neither study provided a systematic assessment of the observational evidence to quantify its magnitude and variation in space and time. Given the dominance of zonally averaged patterns an initial insight into the behavior of isobaric and isothermal heights can be gained from comparing vertical profiles of the zonally averaged pressure and temperature gradients (Fig. 7). We can see that there is a universal pantropical isothermal height around 12 km. At the same time closer to the equator the minimal height where the surface temperature contrasts disappear diminishes (Fig. 7a-c). For comparison, in their model Lindzen and Nigam [1987] adopted a constant isothermal height equal to 10 km. (They assumed that the horizontal temperature differences at the level of $z_{LN} = 3 \text{ km}$ are 30% smaller than the corresponding differences at the sea level: $\Delta T(z_{LN}) = 0.7\Delta T_s$. From $T(z_{LN}) = T_s - \Gamma z_{LN}$ and (14) we obtain $z_i \equiv \Delta T_s / \Delta \Gamma = z_{LN} / 0.3 = 10 \text{ km}$.)

In agreement with Eq. (15), there are two minima of pressure gradients corresponding to two isobaric heights, one closely above the troposphere and another one in the lower atmosphere. The height of the lower minimum grows towards the equator (Fig. 7d-f). As the upper isobaric height is predominantly above the 70 hPa level, we cannot estimate it with certainty in our data. However, there is a tendency for this height to slightly diminish towards the equator.

Generally, a pantropical isobaric (isothermal) height, if it exists, has the following properties: at this height (1) deviation of local pressure (temperature) from the pantropical mean is zero; (2) deviation of local pressure (temperature) from the zonal mean is zero; (3) local horizontal gradient of pressure (temperature) is zero. If a universal isobaric height does not exist, for each grid point these properties can each define a different height. In Fig. 8a,d we plotted zonal averages of the minimal isobaric and isothermal heights calculated according to the above definitions. Local vertical profiles of pressure and temperature differences in individual grid points illustrating how these heights were calculated are exemplified in Fig. 8b,e.

In the majority of grid points there is an isobaric height between 0 and 10 km (Fig. 8c), which corresponds to the lower isobaric height from Eq. (15). It is of interest that the land and the ocean have different lower isobaric heights, with land values peaking below the trade wind inversion layer (3 km) and ocean values peaking at around 6 km (Fig. 8c). As is clear from Fig. 8f, a significant part of grid points has two isothermal heights: one is the pantropical isothermal height around 12 km and the second one is around 3 km.

While our relationship (19) has the limitation of not accounting for the vertical changes in lapse rate, it does provide an insight into the observed behavior of the $\Delta p_s / \Delta T_s$ and $\Delta p_s / \Delta T_a$ ratios. From Fig. 4a we can see that $\Delta p_s / \Delta T_s$ grows with increasing lower isobaric height z_{e1} and, for a given z_e , declines with decreasing isothermal height z_i . As there is a tendency for z_{e1} to grow and for z_i to diminish towards the equator (Fig. 8a,d), this compensating

behavior may explain the approximate constancy of $\Delta p_s/\Delta T_s$ (Fig. 6d). Meanwhile because of the singularity for $z_i \approx 6$ km, a decrease in z_i for $z_e \approx 6$ km, in contrast, leads to a sharp increase in $|\Delta p_s/\Delta T_a|$. This theoretical behavior is consistent with the observed growth of $|\Delta p_s/\Delta T_a|$ in the vicinity of the equator (Fig. 6a).

Fig. 4 also illustrates the sensitivity of these ratios to the values of the lower and upper isobaric heights z_{e1} and z_{e2} . With $z_{e1} \ll z_i$ the ratio of $\Delta p_s/\Delta T_a$ grow approximately proportionally to z_{e1} . Accordingly, land with its significantly lower z_{e1} have lower $\Delta p_s/\Delta T_a$ and $\Delta p_s/\Delta T_s$ than the ocean (Fig. 6a). Thus the observed variability in the lower isobaric height produces uncertainties of the order of 100% in the corresponding estimates of those ratios. While the upper isobaric height appears more conservative, the sensitivity of $\Delta p_s/\Delta T_a$ to its value is much higher. For example, for $z_i = 10$ km a change in z_{e2} of about 10% from 18 km to 20 km diminishes the magnitude of $|\Delta p_s/\Delta T_a|$ by more than an order of magnitude (Fig. 4b).

7 Temporal patterns

We now investigate the time dependence of the relationship between pressure and temperature. If in a certain area we have an isobaric surface at height z_e and an isothermal surface at height z_i , we have (see Eq. 19):

$$\Delta p_s = r \Delta T_a, \quad r \equiv -\frac{z_e}{h_s} \frac{p_s}{T_a} \left(1 - \frac{z_e}{2z_i}\right) \frac{1}{1 - 0.66h_s/z_i} \quad (23)$$

Local values of $\Delta p_s \equiv p_s - \overline{p_s}$ and $\Delta T_a \equiv T_a - \overline{T_a}$ are defined with respect to their mean values in the area where the isobaric and isothermal surfaces exist. Taking the derivative of (23) over time we obtain

$$\frac{\partial \Delta p_s}{\partial t} = r \frac{\partial \Delta T_a}{\partial t} \quad \text{if} \quad \frac{\partial r}{\partial t} = 0, \quad \frac{\partial \Delta X}{\partial t} \equiv \frac{\partial X}{\partial t} - \frac{\partial \overline{X}}{\partial t}, \quad X = p_s, T_a. \quad (24)$$

This means that if z_e , z_i and, hence, r (to the accuracy of a few per cent) do not change with time, temporal changes of Δp_s and ΔT_a are characterized by the same ratio r as their spatial changes.

For each grid point we made a regression of the local monthly changes of sea level pressure, $\tilde{\Delta p}_s(m)$, on the local monthly changes of mean tropospheric temperature, $\tilde{\Delta T}_a(m)$. A similar analysis was performed for p_s and surface temperature T_s . The results are shown in Fig. 9. Here $\tilde{\Delta p}_s(m) \equiv p_s(m) - p_s(m_p)$ and $\tilde{\Delta T}_a(m) \equiv T_a(m) - T_a(m_p)$, where m is the current month and m_p is the previous month (e.g., January and December). Thus $\tilde{\Delta p}_s(m)/\tilde{\Delta m}$ and $\tilde{\Delta T}_a(m)/\tilde{\Delta m}$, where $\tilde{\Delta m} = 1$ month, represent the monthly mean temporal derivatives of sea level pressure and tropospheric temperature in a given grid point. Linear regression of $\tilde{\Delta p}_s$ on $\tilde{\Delta T}_a$ characterizes the annually averaged dependence between monthly changes of pressure and temperature which correspond to Eq. (24) when $\partial \overline{p_s}/\partial t = 0$ and $\partial \overline{T_a}/\partial t = 0$. As we can see from Fig. 9, the pantropical mean monthly changes of sea level pressure and air temperature are significantly smaller compared to the majority of the corresponding local changes, so conditions $\partial \overline{p_s}/\partial t = 0$ and $\partial \overline{T_a}/\partial t = 0$ approximately hold for the tropics as a whole.

Our analysis shows that in the equatorial land regions with high rainfall – in the Amazon and Congo river basins, see point C in Fig. 9 – the regressions were not significant at 0.01 probability level³. Where the regressions are significant, the largest (by absolute magnitude)

³On land, sea level pressure is not an empirically measured variable, but is calculated from pressure $p_l(z_l)$, temperature $T_l(z_l)$ and the geopotential height z_l of the land surface assuming $\Gamma = 6.5$ K km⁻¹ for $0 \leq z \leq z_l$, where $z = 0$ corresponds to the sea level. This definition introduces a formal dependence of p_{sl} (sea level pressure on land) on surface air temperature T_l , the strength of which is directly proportional to z_l .

regression slopes tend to be concentrated in the regions of the largest meridional gradients of sea level pressure, i.e. around the 15 and 20 degrees latitudes (Fig. 9). These local dependences between $\tilde{\Delta}p_s$ and $\tilde{\Delta}T_s$ can be explained by the seasonal migration of the Hadley cells where lower pressure is spatially associated with higher temperature (see Fig. 10).

This explanation agrees with the observation that in those extratropical regions where areas of low pressure are at the same time areas of low temperature (particularly the southern Ferrel cell), the seasonal relationship between pressure and temperature changes is generally less consistent than it is in the tropics with local relationships occasionally being reversed – i.e., pressure and temperature rise or decline together (see point E in Fig. 9). Local changes in p_s and T_a appear affected by migration of the circulation cells with a spatially invariable temperature-pressure relationship within the cells.

Plotting local absolute changes of p_s and T_a instead of local changes relative to the tropical mean in Fig. 9 has the advantage that the resulting figure does not depend on the averaging domain (cf. Fig. 6a) and allows for comparison of tropical and extratropical patterns. However, we additionally performed a regression of local relative monthly changes as in Eq. (24) for each grid point between 22.5°S to 22.5°N. The tropical mean values (\pm standard deviation) of the obtained local slope coefficients are as follows. For the relationship between p_s and T_a : -2.4 ± 1.8 hPa K⁻¹ (-2.5 ± 0.8 hPa K⁻¹ for the land, -2.3 ± 1.9 hPa K⁻¹ for the ocean). For the relationship between p_s and T_s : -0.9 ± 0.6 hPa K⁻¹ (-1.2 ± 0.6 hPa K⁻¹ for the land, -1.1 ± 0.6 hPa K⁻¹ for the ocean). We can see that these figures are again very close to the corresponding spatial ratios (Fig. 5a, Fig. 6a,d). There is, however, less difference between the land and the ocean than in the spatial ratios.

In their analysis of the observed long-term p_s and T_a changes Bayr and Dommenges [2013] compared relative partial pressures and relative tropospheric temperatures in 1989-2010 (their Fig. 12). They found that, in agreement with Eq. (24), these changes are related by practically identical ratios, -2.0 hPa K⁻¹ for land, -2.4 hPa K⁻¹ for ocean, and -2.3 hPa K⁻¹ for tropics as a whole, as the corresponding spatial contrasts shown in Fig. 5a. This pattern is not preserved in the IPCC multimodel ensemble [Bayr and Dommenges, 2013, their Fig. 5]: modelled long-term changes for the time period 1970-2099 are characterized by a lower ratio for land (-2.5 hPa K⁻¹) than for the ocean (-1.9 hPa K⁻¹) with an overall mean of -2.0 hPa K⁻¹.

8 Discussion

We have critically examined the model of Bayr and Dommenges [2013]. For an atmosphere where the lapse rate is everywhere the same, the correct expression for the dependence between sea level pressure and tropospheric temperature is given by Eq. (20): it differs from Eq. (3) of Bayr and Dommenges [2013] by the absence of $1/2$ and the presence of surface air density ρ_s instead of mean tropospheric density ρ_a . Eq. (20) is similar to Eq. (3) in that it describes a direct proportionality between the isobaric height z_e and the $\Delta p_s / \Delta T_a$ ratio.

In the real atmosphere the lapse rate varies considerably in the horizontal plane: the lapse rate over the warmer surfaces is on average steeper than over the colder surfaces (Fig. 2). We have shown that in such an atmosphere there must be at least two isobaric heights $z_{e1} \leq z_{e2}$ (Eq. 15, Fig. 3c). The models of Lindzen and Nigam [1987] and Bayr and Dommenges [2013] each considered only one isobaric height, while the existence of the second one and its link to the first was not discussed. In agreement with our Eq. (15), observations show that

That is, p_{sl} diminishes with growing T_l even if p_l and, hence, the amount of gas in the atmospheric column remains constant. Approximating the hydrostatic equation (5) as $(p_l - p_{sl})/z_l = -p_l/h$, $h = RT_l/(Mg)$, and taking the derivative of this equation over T_l at constant p_l we obtain $dp_{sl}/dT_l = (z_l/h)p_l/T_l$. For the mean geopotential height $z_l = 0.6$ km of the tropical land, $p_l = 950$ hPa and $T_l = 295$ K we find $dp_{sl}/dT_l = -0.2$ hPa K⁻¹, i.e. about 20% of the mean ratio established by us for the tropical land (Fig. 9) is not related to any air redistribution but is a formal consequence of the definition of p_{sl} .

in the tropical atmosphere the two isobaric heights correspond to $z_{e1} \sim 1.5 - 6$ km and $z_{e2} \sim 17 - 20$ km. Both heights are spatially variable (Fig. 8a-c). Observations do not support the existence of a pantropical constant isobaric height either around 3 km as in the model of Lindzen and Nigam [1987] or at the top of the troposphere as in the model of Bayr and Dommenges [2013]. Apparently, in the presence of two isobaric heights the $\Delta p_s / \Delta T_a$ ratio cannot be a linear function of z_e . Indeed, we have shown that there is an additional essential parameter governing the relationship between pressure and temperature, the isothermal height z_i . The resulting dependence of $\Delta p_s / \Delta T_a$ on the isobaric height is quadratic, not linear (Eq. 19).

Isothermal height z_i characterizes the thermal structure of the troposphere. In the limit of very large z_i the ratio $\Delta p_s / \Delta T_s$ (and $\Delta p_s / \Delta T_a$) do not depend on z_i , but only on isobaric height z_e (Eqs. 16, 20). If z_e is given, Δp_s depends only on surface temperature contrasts. This fact apparently facilitated interpretation of surface pressure contrasts as *determined* by surface temperature [Lindzen and Nigam, 1987, Sobel and Neelin, 2006, An, 2011]. For example, Sobel and Neelin [2006, p. 324] in their discussion of the model of Lindzen and Nigam [1987] noted that surface temperature determines temperature in the atmospheric boundary layer, which, in turn, determines surface pressure via a hydrostatic relationship. Indeed, if the lapse rate does not vary in the horizontal plane, $z_i = +\infty$ and there is a direct proportionality between Δp_s and ΔT_s (and ΔT_a). But in the real atmosphere z_i is relatively large not because the release of latent heat in the troposphere does not matter. On the contrary, it is the latent heat release that works to elevate the isothermal height by diminishing the difference in the mean tropospheric lapse rate between the warmer and colder surface areas that could have otherwise been much larger (Fig. 2). An illustration is shown in Fig. 11 which compares pressure and temperature profiles in Sahara and East China in July each with the zonal mean profile. One can see that in the dry Sahara the isothermal height is very small while in East China (where it is the monsoon period) it is on average much higher. On the other hand, when z_i is not very large but comparable to z_e , it has a crucial impact on the relationship between pressure and temperature (Eq. 19).

We have shown that the character of the relationship between $\Delta p_s / \Delta T_a$ (as well as of $\Delta p_s / \Delta T_s$) is extremely sensitive to the values of z_e and z_i in the interval of their observed values (Fig. 4). This sensitivity has not been previously explored. For example, at $z_i = 10$ km, which is the value adopted by Lindzen and Nigam [1987], a 15% change in the upper z_e from 17 km to 20 km leads to a complete disappearance of the dependence of Δp_s on both ΔT_s and ΔT_a . Therefore, the assumed approximate constancy of the upper isobaric height z_{e2} , which motivated the model of Bayr and Dommenges [2013], proves to be too poor a representation of reality for this height to serve as a determinant of the $\Delta p_s / \Delta T_a$ ratio. The sensitivity to the lower isobaric height z_{e1} is lower: at constant z_i the ratio $\Delta p_s / \Delta T_a$ is directly proportional to z_{e1} (Fig. 4b). However, data show that z_{e1} experiences a proportionally higher spatial variation than z_{e2} . It varies from a few hundred meters over land to over 6 km over the ocean (Fig. 8c). In the result, z_{e1} appears as invalid for a theoretical prediction of $\Delta p_s / \Delta T_a$ as is z_{e2} . With several different peaks, different values for land and ocean and mean values depending on latitude (Fig. 8a,d) the isobaric and isothermal heights can hardly be specified from some independent physical considerations. Rather, they are themselves dictated by the dynamic relationships between pressure and temperature.

We have emphasized an important property of the isobaric height: it links sea level pressure contrasts to the pressure contrasts in the troposphere (Eq. 22). If the isobaric height is unknown, so are the surface pressure contrasts. Unlike the tropospheric pressure contrast, the surface pressure contrasts cannot be determined from consideration of temperature gradients alone. This summarizes a major problem for the theory of atmospheric circulation. Having set a temperature gradient, one can easily find tropospheric pressure gradients and, consequently, the geostrophic winds in the troposphere. However, what type of circulation can be generated in the low-level atmosphere remains uncertain. To simulate low-level winds generated by differential heating, one has to specify the dynamic interaction between the up-

per and lower atmosphere or, simply put, the turbulent friction. Depending on the adopted parametrization, one and the same temperature gradient can be modelled to produce drastically different low-level winds. E.g., for an axisymmetric general atmospheric circulation different assumptions regarding friction yield diverse results from complete absence of any low-level circulation to negligible meridional circulation to circulations close to the observed [e.g., Held and Hou, 1980, Schneider, 2006]. Through parametrization of turbulence, models of more local circulations driven by differential heating adopt as granted the basic parameters of the larger-scale circulations into which they are embedded [e.g., Smagorinsky, 1953, Pielke, 1981, Curry, 1987]. While optimized to provide an adequate description of the relevant processes, such models would yield very different results if those basic parameters change.

A simple but relevant illustration to these ideas is provided by the model of Lindzen and Nigam [1987]. They investigated how low-level winds depend on the isobaric height z_e in their model, where the boundary layer height was assumed to be equal to z_e . In the limit $z_e \rightarrow 0$ the surface pressure gradients disappear (Eq. 16) and the low-level winds should vanish. Contrary to this expectation Lindzen and Nigam [1987] found little dependence of the meridional winds (and moisture convergence) on z_e in their model. A smaller z_e expectedly produced weaker surface pressure gradients, but it also produced a proportionally larger damping coefficient $\epsilon \equiv C_D |V_c| / z_e$, where C_D is a constant and V_c is a typical wind speed at z_e taken to be equal to 8 m s^{-1} . As a result of a weaker meridional pressure gradient, the zonal wind did decrease proportionally to the surface pressure gradient. However, the meridional wind proportional to the product of zonal wind and the damping coefficient ϵ [Lindzen and Nigam, 1987, see their Eq. 12a] did not change much. Here the decrease in pressure gradient was offset by an increase in the damping coefficient ϵ , such that the low-level air convergence remained approximately independent of z_e (and, hence, of surface pressure gradients).

This independence resulted from how friction was parameterized, in particular, from the assumed constancy of V_c and, hence, from the inverse proportionality between the damping coefficient and the isobaric height. In the real atmosphere the height of boundary layer h_b is much smaller than the isobaric height, $h_b \sim 1 \text{ km} \ll z_e$, especially over the ocean (Fig. 8c). Because of this, pressure gradients at the top of the boundary layer are determined by the surface pressure gradients and close to them. Since at the top of the boundary layer winds are approximately geostrophic [Back and Bretherton, 2009], this means that the geostrophic wind speed V_c at the top of the boundary layer (which is used in the determination of the damping coefficient) is approximately proportional to the surface pressure gradient. Consequently, it must decrease with decreasing z_e . In the result, with decreasing z_e (decreasing surface pressure gradient), surface winds and moisture convergence should decline as well. On the other hand, z_e can be itself a function of pressure gradients. This simple example illustrates that our answer to the question: "what happens in the lower atmosphere" is fully determined by how turbulence is parameterized and is practically independent of the magnitude of temperature contrasts. The opposite is true for the geostrophic wind in the upper atmosphere. Since turbulent friction is directly related to the dissipative power of atmospheric circulation, finding constraints on this power will help resolve the big challenge of predicting low-level circulation and moisture convergence [Makarieva et al., 2013a].

We have seen that most part of the relationship between p_s and T_a in the tropics shown in Fig. 3 of Bayr and Dommenges [2013] can be attributed to the properties of the tropical atmosphere around 20 degrees latitudes and in the region of the Walker circulation (Fig. 5d) as well as to its zonally averaged properties (Fig. 6c). This means that to explain why the ratios between sea level pressure and temperature contrasts have their observed magnitudes we have to explain why the Hadley and Walker circulations are characterized by those pressure and temperature contrasts as they are. Other processes, including seasonal variation and the observed long-term relative p_s and T_a changes as in Fig. 12 of Bayr and Dommenges [2013], appear to preserve the vertical structure of the atmosphere set by the main dynamic drivers of the circulation. In other words, if we knew why there is a $\sim 10 \text{ hPa}$ pressure contrast per

$\sim 10^\circ\text{C}$ tropospheric and surface temperature contrast in Hadley cells (Fig. 10) we would be able to determine the isobaric heights and thus understand why the tropical troposphere is about 16 km and not, say, 10 or 25 km high as well as why a local temperature increase leads to a local sea level pressure drop of a given magnitude. Two drivers of low-level circulation have been considered [Gill, 1980, Lindzen and Nigam, 1987, Neelin, 1989, Sobel and Neelin, 2006, Back and Bretherton, 2009, An, 2011]: surface heating and the release of latent heat. A distinct physical process was recently described. Horizontal transport of moisture with its subsequent condensation and precipitation away from the point where it evaporated produces pressure gradients due to the changing concentration of water vapor as the air moves from the evaporation to condensation area [Makarieva et al., 2013b, 2014]. Pressure is greater where water vapor is added and lower where it is removed from the air column. Understanding the relative contributions of these processes will guide our predictions of local pressure and circulation changes. We believe that the analysis of pressure/temperature relationships initiated by Bayr and Dommenget [2013] can shed light on the relative strength of the contributing processes if these are studied together with the moisture contrasts.

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Relationship between T_s and T_a

The relationship between surface temperature T_s and the mean temperature $T_a(Z)$ of the atmospheric column below Z can be derived from (9) and the hydrostatic equation (5):

$$T_a(Z) \equiv \frac{\int_0^Z T(Z) \rho dZ}{\int_0^Z \rho dZ} = \frac{T_s}{1+c} \frac{1 - e^{-cZ} e^{-Z} e^{-cZ^2/2}}{1 - e^{-Z} e^{-cZ^2/2}}, \quad Z \equiv \frac{z}{h_s}, \quad cZ \ll 1. \quad (25)$$

Expanding (25) over c and keeping the linear term we have

$$T_a = T_s \left[1 - c \left(1 - \frac{Z}{e^Z - 1} \right) \right]. \quad (26)$$

Taking the derivative of (26) over T_s and c we obtain:

$$dc = \frac{db - dn}{1 - Z/(e^Z - 1)}, \quad dn \equiv \frac{dT_a}{T_a}. \quad (27)$$

For the height of the tropical troposphere $z = H = 16.5$ km, $T_s = 298$ K and $\Gamma = 6.1$ K km⁻¹ (Fig. 2) we have $Z = 1.9$, $c = 0.18$, $1 - Z/(e^Z - 1) = 0.66$ and obtain from (26) and (27)

$$T_a = 0.88T_s, \quad db = dn + 0.66dc, \quad \frac{dT_s}{dT_a} = \frac{1}{0.88} \left(1 + 0.66 \frac{d\Gamma}{dT_a} \frac{T_a}{\Gamma_g} \right). \quad (28)$$

The mean tropospheric $\overline{T_a} = 262$ K in the tropics estimated from (28) is identical to the annual tropical mean $\overline{T_a} = 262$ K (22.5°S – 22.5°N) that we estimate from the TTT data of Mears and Wentz [2009] and close to $\overline{T_a} = 263.6$ K cited by Bayr and Dommenget [2013].

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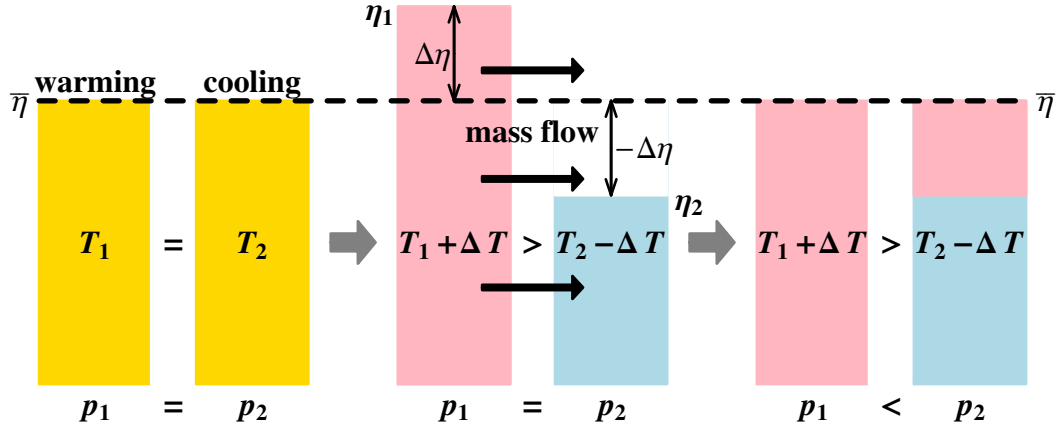


Figure 1: Schematic of the physical model of Bayr and Dommenges [2013] with a constant mean column height $\bar{\eta}$ (re-drawn from Fig. 4 of Bayr and Dommenges [2013]). To balance two columns of heights η_1 and η_2 , one should either move to the second column from the first *half* of the total height difference $\eta_1 - \eta_2$ or the *entire* difference $\Delta\eta \equiv \eta_1 - \bar{\eta}$ between η_1 and the mean height $\bar{\eta} \equiv (\eta_1 + \eta_2)/2$.

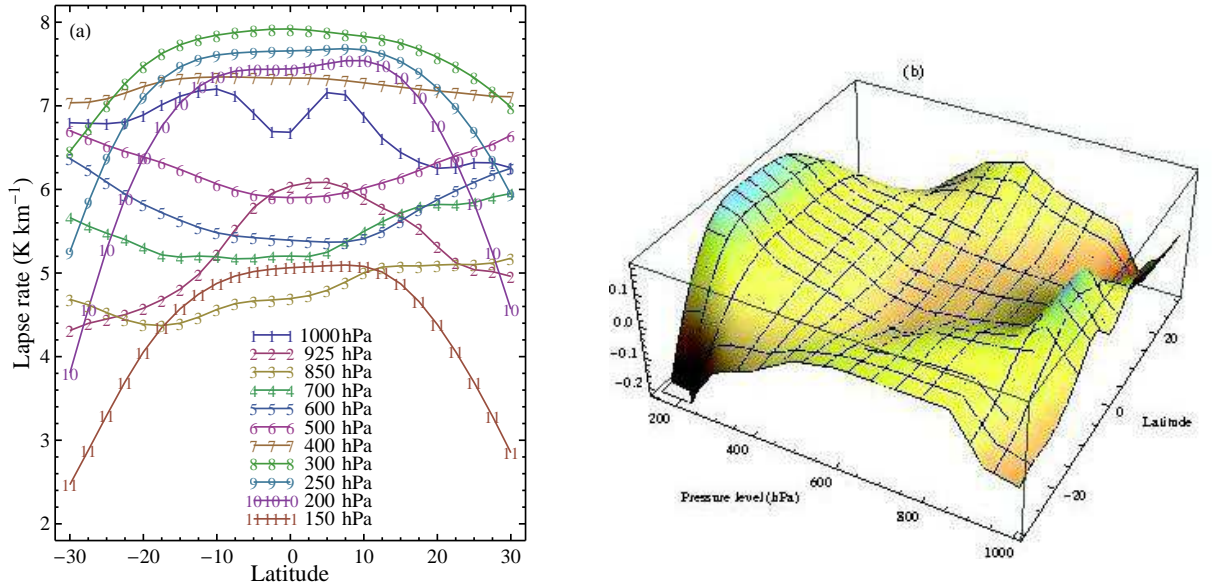


Figure 2: Annual mean latitudinal profiles of the air temperature lapse rate on different pressure levels. For example, curve 1 in (a) shows the mean lapse rate between 1000 hPa and 925 hPa; curve 11 – between 150 and 100 hPa. The tropical mean lapse rate (the temperature difference between 1000 hPa and 100 hPa levels divided by the difference in the geopotential heights and averaged from 30°S to 30°N) is 6.0 K km⁻¹. Panel (b) shows the relative variation – at each pressure level the lapse rate at a given latitude is divided by the mean lapse rate at this level (averaged between 30°S and 30°N). The equator has a higher lapse rate than the 30 degrees latitudes in the lower and upper – but not the middle – troposphere.

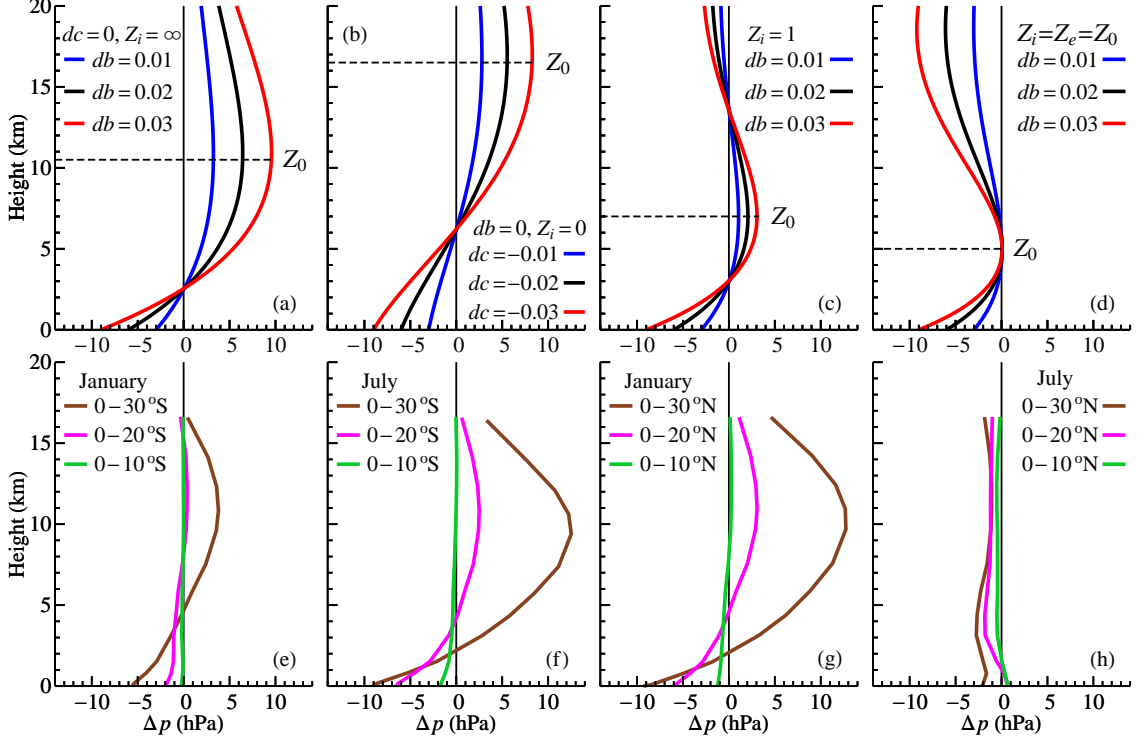


Figure 3: Vertical profiles of pressure differences $\Delta p(z)$ between air columns differing in their lapse rate, surface pressure and temperature. Panels (a)-(d): theoretical profiles (12) with $dp = \Delta p$, $da = \Delta p_s/p_s$, $db = \Delta T_s/T_s$, $dc = \Delta \Gamma/\Gamma_g$ (cf. 13), $p_s = 1000$ hPa, $T_s = 300$ K. In panels (a)-(d) $da = -0.003, -0.006, -0.009$ for the blue, black and red curves, respectively. In each panel $da/db = \text{constant}$ for all the three curves. Dashed line Z_0 (21) shows the height where the positive pressure difference in the upper atmosphere is maximum, $\Delta p(Z_0) = \Delta p_0$ (22). Note two isobaric heights in panel (c). In panel (d) condition $Z_i = Z_0$ (the atmosphere is horizontally isothermal where the positive pressure difference aloft is maximum) yields $Z_i = Z_0 = Z_e = -2da/db = (-2da/dc)^{1/2}$, see (15), (21) and (14), and $\Delta p_0 = 0$, i.e. the pressure surplus aloft disappears. Panels (e)-(h): real vertical profiles of zonally averaged pressure differences between the air columns at the equator and 10, 20 and 30 degrees latitudes in the Southern (e,f) and Northern (g,h) hemispheres in January (e,g) and July (f,h). E.g., the brown line in (e) shows the difference between the air column at the equator and at 30°S in January. Note that while the theoretical curves (a-d) in each panel are chosen such that they have one and the same isobaric height Z_e (i.e., they cross the line $\Delta p = 0$ at the same point), this varies for the real profiles (e-h).

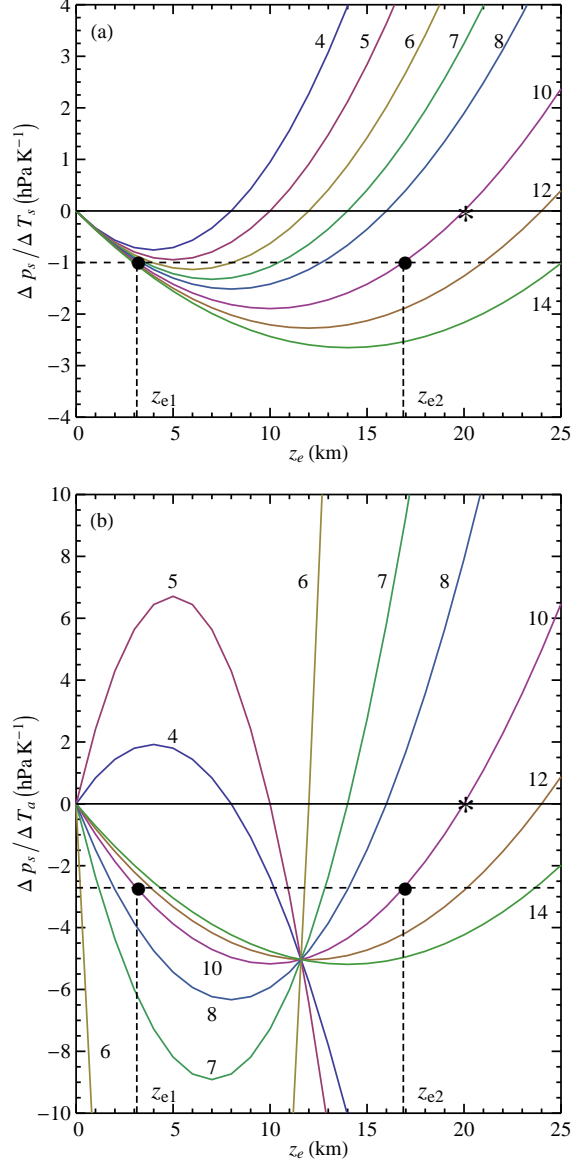


Figure 4: Dependence of $\Delta p_s / \Delta T_s$ (a, Eq. 16) and $\Delta p_s / \Delta T_a$ (b, Eqs. 19, 23) on isobaric height z_e for different values of isothermal height z_i (km) that are shown near the corresponding curves. Vertical dashed lines denote isobaric heights z_{e1} and z_{e2} corresponding to the tropical mean $\Delta p_s / \Delta T_s = -1$ hPa K⁻¹ (horizontal dashed line in (a), Fig. 6d) for $z_i = 10$ km. With an increase in z_{e2} from 17 km to 20 km for $z_i = 10$ km both $\Delta p_s / \Delta T_s$ and $\Delta p_s / \Delta T_a$ become zero (shown as asterisk).

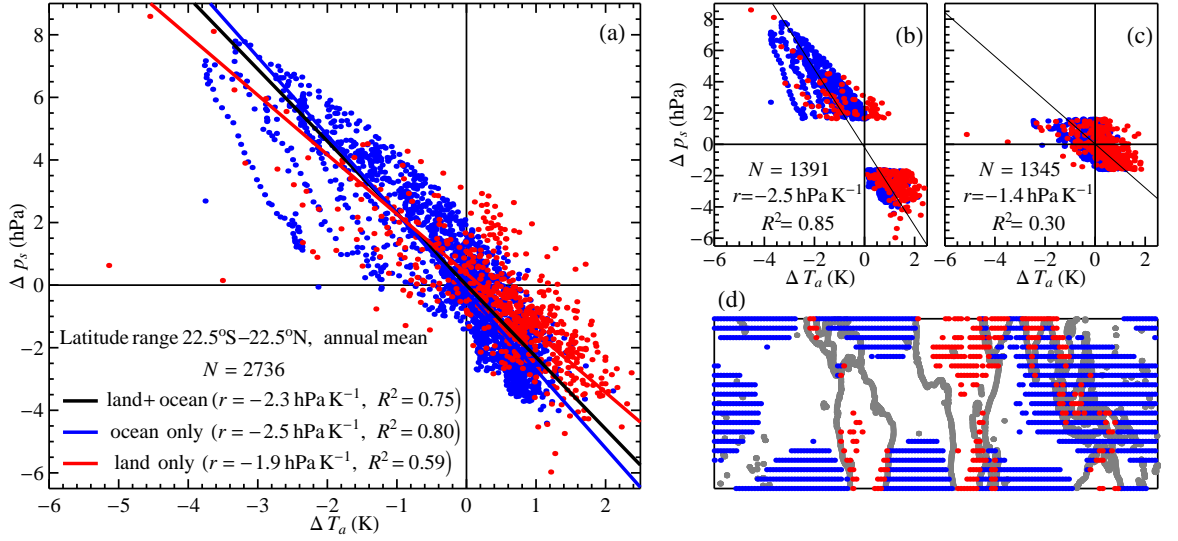


Figure 5: Dependence of local sea level pressure p_s on local mean tropospheric temperature T_a . (a) Regression slopes r and squared correlation coefficients R^2 for the linear regression $\Delta p_s = r \Delta T_a$, where $\Delta p_s = p_s - \overline{p_s}$ and $\Delta T_a = T_a - \overline{T_a}$, where all local values are annual means and the overbars denote averaging over the latitude range 22.5°S and 22.5°N, for land (red), ocean (blue) and area as a whole (black). (b): r and R^2 for regression of data points where $|\Delta p_s|$ is greater than 1.6 hPa, which is equal to 2/3 standard deviation of the frequency distribution of Δp_s values in (a). (c): The same as (b) but for data points where $|\Delta p_s|$ is smaller than 1.6 hPa. (d): Geographic location of the grid points from panel (b). Empty space corresponds to grid points from panel (c).

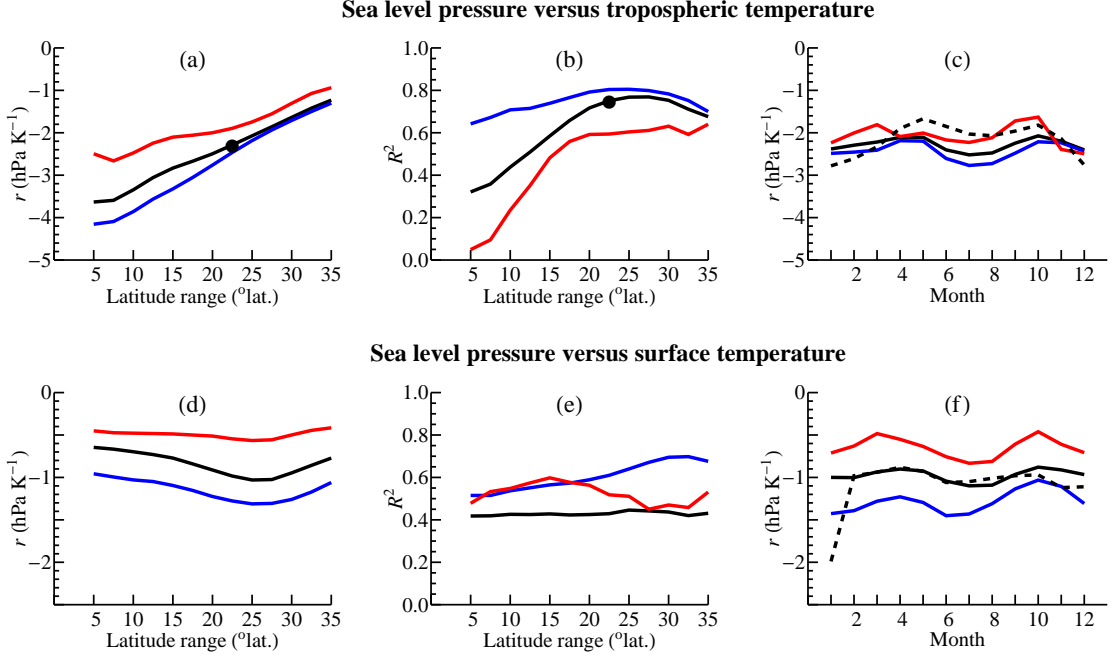


Figure 6: (a,b) and (d,e): Regression slopes r and R^2 for the regression $\Delta p_s = r\Delta T_a$ (a,b) and $\Delta p_s = r\Delta T_s$ (d,e) for land (red), ocean (blue) and total area (black) as dependent on the considered latitude range. E.g. latitude range 35°lat. means that averaging is made from 35°S to 35°N . Here $\Delta X \equiv X - \overline{X}$ ($X = p_s, T_a, T_s$) where X is the local annual mean value and \overline{X} is its spatial average over the considered latitude range. Data corresponding to Fig. 3 of Bayr and Dommenget [2013] (and to our Fig. 5a) are marked with black circle. (c,f) Seasonal variation of r for the latitude range 22.5°S and 22.5°N : land (red), ocean (blue) and total tropics (black) curves denote results as in panels (a,d) but for particular months; the dashed line denotes the regression slope of zonally averaged Δp_s and ΔT_a (ΔT_s).

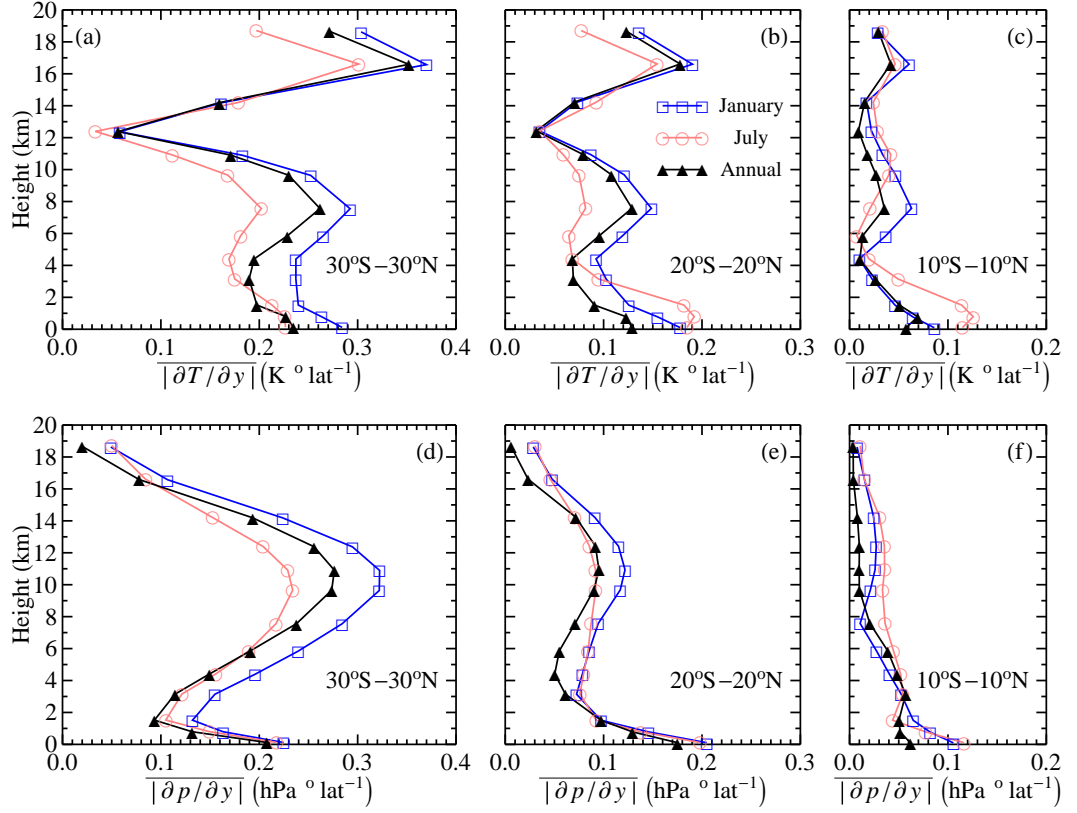


Figure 7: Vertical profiles of the meridional temperature (a-c) and pressure (d-f) gradients taken by absolute magnitude and averaged from 30°S to 30°N (a,d), 20°S to 20°N (b,e) and 10°S to 10°N (c,f) in January (blue squares), July (pink circles) and annually (black triangles).

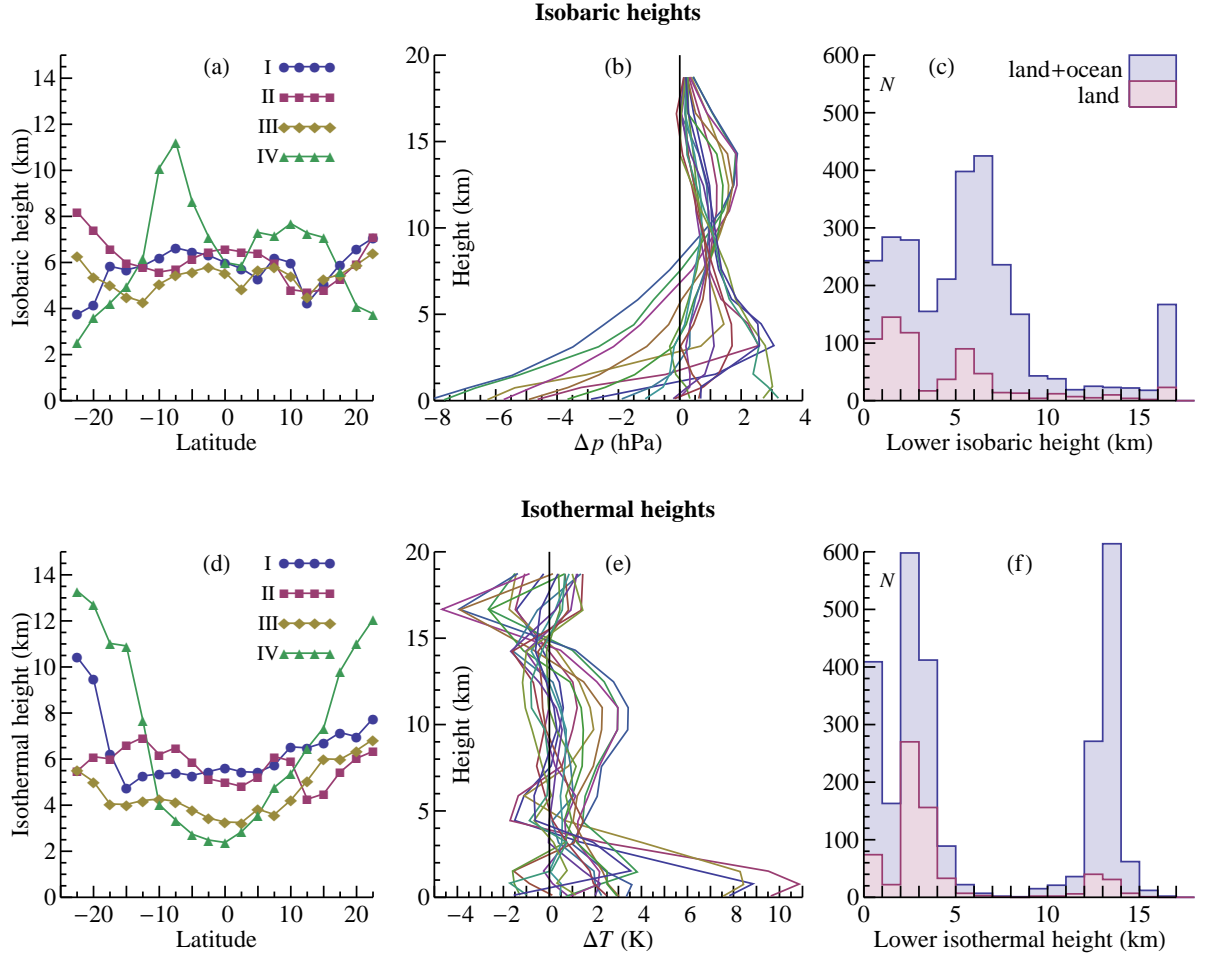


Figure 8: Isobaric (a-c) and isothermal (d-f) heights in the tropical troposphere. (a),(d): zonal averages of isobaric and isothermal heights defined as follows: I – minimum height where local pressure (temperature) coincides with the mean pantropical pressure (temperature) at this height, II – minimum height where local pressure (temperature) coincides with the mean zonal pressure (temperature) at this height, III – minimum height where local zonal gradient of pressure (temperature) is zero, IV – minimum height where local meridional gradient of pressure (temperature) is zero. All local pressure and temperature values are annual means. (b), (e): vertical profiles of the differences between the mean pantropical profile of pressure (temperature) in July and the profile of pressure (temperature) for 18 individual grid points at 20°N that are spaced by 20° longitude starting from 0°E. Note the difference between isobaric heights I and II: individual profiles coincide with each other at a different height (around 10 km) than they coincide with the pantropical mean profile. (c), (f): frequency distribution of isobaric and isothermal heights 1, the inner histogram shows the distribution of land values only and the outer histogram shows all values.

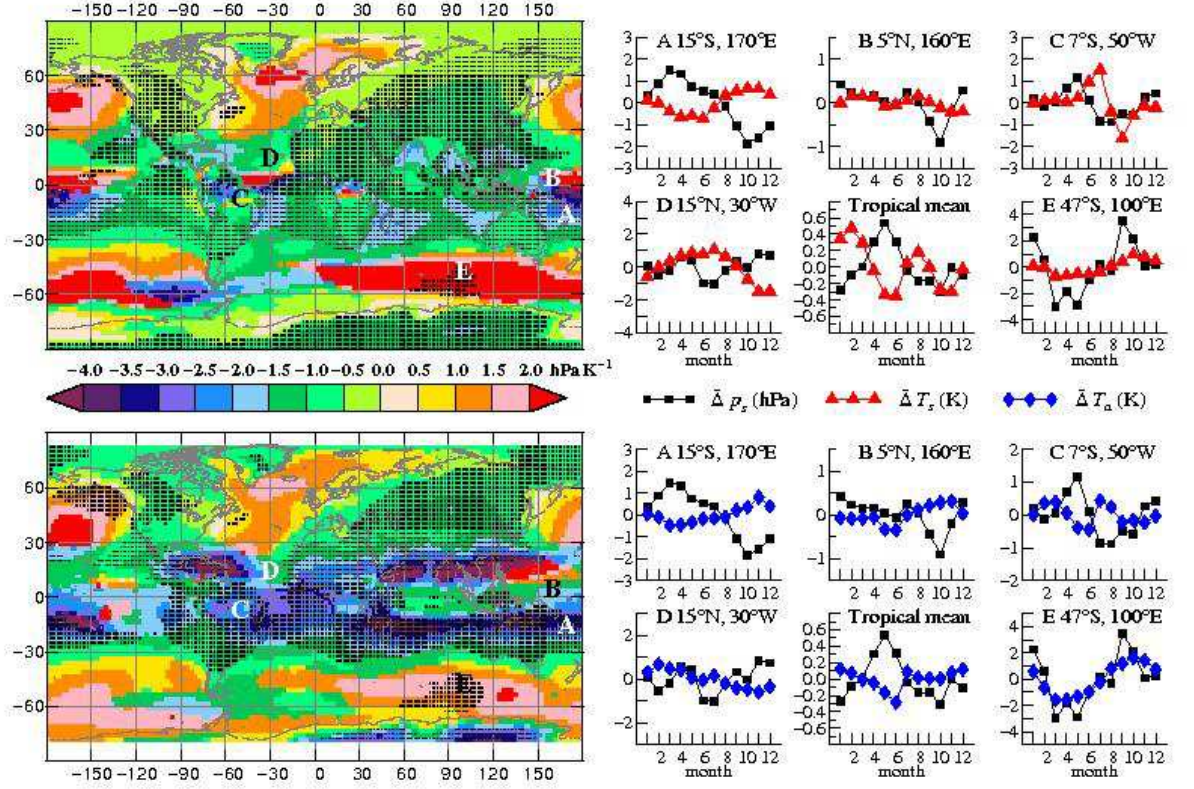


Figure 9: Mean ratio between local monthly changes of sea level pressure p_s and surface temperature T_s (larger top left panel) and p_s and tropospheric temperature T_a (larger lower left panel). The ratio is estimated as the slope coefficient of a Reduced Major Axis regression of $\tilde{\Delta} p_s$ on, respectively, $\tilde{\Delta} T_s$ and $\tilde{\Delta} T_a$, see Section 7 for details. Black dots indicate where the probability level of the regression is less than 0.01. The small panels exemplify seasonal changes of $\tilde{\Delta} p_s$, $\tilde{\Delta} T_s$ and $\tilde{\Delta} T_a$ in individual grid points (A, B, C, D and E) shown in the big panels, as well as the tropical mean (the area between 22.5°S and 22.5°N). Note the different vertical scales in the small panels.

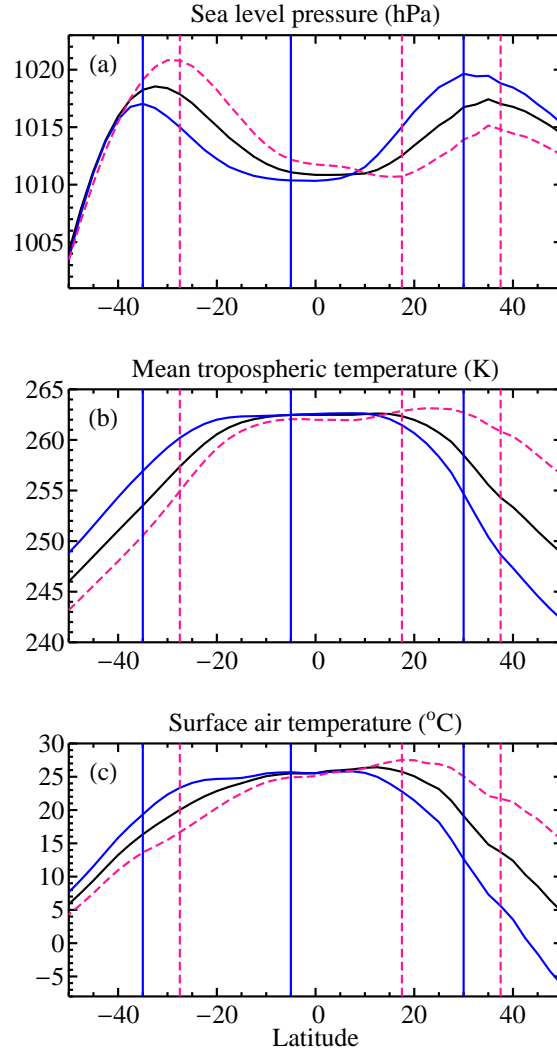


Figure 10: Zonally averaged atmospheric parameters of Hadley cells. Solid black curve: annually averaged data, solid blue curve: January, dashed pink curve: July. Vertical lines show the borders of the Southern and Northern cells in January (solid blue) and July (dashed pink) defined as the poleward maxima (the outer borders) and the central minimum (the inner border) of sea level pressure (a). Monthly data from NCAR-NCEP reanalysis averaged for 1978-2013.

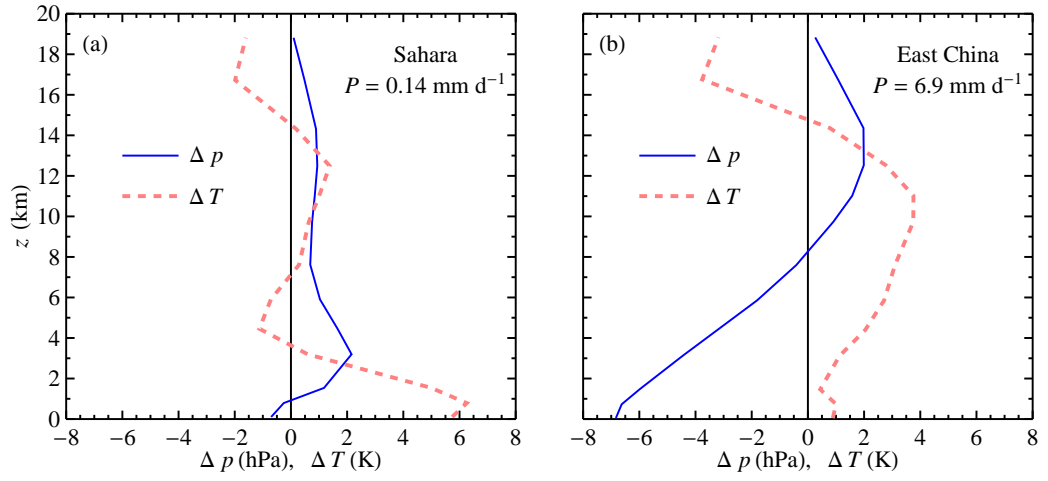


Figure 11: Vertical profiles of air pressure and temperature differences between the zonal mean (20°N - 30°N) in July and (a) Sahara (20°N - 30°N, 0°E - 20 °E) and (b) East China (20°N - 30°N, 100°E - 120 °E). P is precipitation in July in these regions.